Short Positions, Rally Fears and Option Markets

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Abstract

Index option pricing on world market indices are investigated using Lévy processes with no positive jumps. Economically this is motivated by the possible absence of longer horizon short positions while mathematically we are able to evaluate for such processes the probability of a Rally Before a Crash (RBC). Three models are used to effectively calibrate index options at an annual maturity and it is observed that positive jumps may be needed for FTSE, N225 and HSI. RBC probabilities are shown to have fallen by 10 points after July 2007. Typical implied volatility curves for such models are also described and illustrated. They have smirks and never smile.

1 Introduction

This paper examines the pricing of index options across a number of international equity indices using jump diffusion models with no positive jumps. We have modeled price processes for some ten years now using processes with two sided jumps and we cite as examples Carr, Geman, Madan and Yor (2002), Eberlein and Prause (2002). However, the maturity spectrum of traded options has expanded considerably and currently on major indices we have over 200 options trading with maturities ranging from 3 to 12 years. We focus attention here on the structure of Lévy processes consistent with the marginal stock price distributions extracted from the longer maturity options. In this regard there are some considerations in support of such an asymmetric modeling choice.

First, we note that economically it has long been recognized that down side put options implicitly value the crash fears of market participants who have a long position in the underlying index (Bates (2000)). From general equilibrium considerations we know that the aggregate economy must be long the stock indices at all maturities and so this crash protection premium is present across the option surface. Symmetrically one also has the value of rally fears of participants who are short the index, embedded in the upside call options. Empirically it has been observed that measure changes consistent with market option prices are U-shaped (Jackwerth and Rubinstein (1996), Jackwerth (2000), Carr, Geman, Madan and Yor (2002)). These observations led Bakshi and Madan (2007) to derive U-shaped measure changes in an equilibrium with heterogeneous agents some of whom are short the market index. Empirical investigations on one month maturity options, in the direction of U-shaped kernels have been further pursued by Bakshi, Madan and Panayotov (2009). Nonetheless, these latter fears of a market rally may be sufficiently curtailed in financial markets by the absence of significant short positions particularly at the longer option maturities. Furthermore the financing of longer term downside protection by the writing of covered calls also tends to dampen the upward smile or premiums. Hence for a variety of reasons long maturity stock price distributions embedded in option prices may well be consistent with processes with no positive jumps. We are not arguing that upward jumps do not occur under the true measure, but merely enquiring whether long maturity risk neutral distributions are possibly consistent with processes of this type.

Additionally we observe a growing interest in such processes especially in the credit arena where the emphasis is on downward moves and one has access to first passage time distributions for such processes. In this regard we cite Rogers (2000), Lipton (2002) and Madan and Schoutens (2008). These computations are involved when working with finite horizons, and require two dimensional inversions of Laplace transforms. Somewhat simpler are calculations of certain probabilities. For example we note that for such processes, called spectrally negative (Bertoin (1996)), we may easily compute the probability of a $y\%$ rally before an $x\%$ crash. The extraction of such information from market option prices on the calibration of a spectrally negative process could serve as an economic indicator of general market interest. We call this probability the Rally Before Crash (RBC) probability. One may get these probabilities by simulation but this is quite expensive computationally. An analytical computation is much preferred. The finite horizon version of these probabilities are actively traded in foreign exchange markets under names like digital double no touch, or touch/no touch. The simpler computations are for the perpetual probabilities presented here. We agree that the daily option surface is potentially rich in information content, but expect that the most useful information probably lies in suitable summary transformations that prove to yield explanatory variables of some predictive significance. It is with such a motivation that we consider the RBC probability. We might ask whether such a variable is a leading indicator for when the market bottom is behind us and the immediate future is upwards. An investigation in this direction requires the construction of RBC probabilities at various maturities as inputs for a more extended study. The probabilities could also be used to fine tune the hedging of down side risk.

From an empirical perspective we further note that longer maturity index implied volatility curves as presented for example in Broadie, Chernov and Johannes (2007), display a smirk but no smile. We shall observe later that such implied volatility curves are a characteristic feature of a process with no positive jumps.

Such spectrally negative processes or processes with no positive jumps have appeared in the literature and a case in point is the finite moment log stable process of Carr and Wu (2003). Their motivation was however related to the term structure of implied volatility skews and a related need to entertain infinite variance models for the logarithm of the stock price. We entertain instead, for the logarithm of the stock price, a finite variance spectrally negative Lévy process that extends the original Black and Scholes (1973) and Merton (1973) geometric Brownian motion model to a jump diffusion model with no positive jumps that we shall call a negative jump diffusion model.

Such a negative jump diffusion model is specified on choosing the jump measure. We employ three jump measures in our study. The first is just the negative part of the jump measure of the $CGMY$ model of Carr, Geman, Madan and Yor (2002). We call this model CGY as one has effectively set M equal to infinity. Recently, Kyprianou and Rivero (2007) have introduced two four parameter variations of CGY that yield explicit solutions for the RBC probability in the absence of a diffusion. As we admit a diffusion, we shall obtain the RBC probability by efficient numerical methods. We include in our study these two models of Kyprianou and Rivero (2007). These models are based on two special conjugate Bernstein functions and we call the models KR and its conjugate KRC.

Each of the three models CGY, KR, KRC are calibrated once a month for one year to each of seven index options SPX, $FTSE$, $EUROSTOXX$, $N225$, $GDAXI$, HSI, and $IBEX$ at approximate maturities of a quarter, a half year and a year. The reason for calibrating separately at each maturity is that the processes used are Lévy processes and it is known (Konikov and Madan (2000)) that such processes fit well at each maturity but not so well across maturities. Additionally we note that RBC probabilities may currently only be evaluated for Lévy processes. The particular choice of a quarter, a half year and a year is arbitrary, but they are typical horizons of interest.

We find that negative jump diffusion models perform relatively poorly on the $FTSE$, HSI and $N225$. This leads us to conjecture that perhaps these markets have a greater exposure to the fear of a rally. Surprisingly, for us, the other markets are well fitted by negative jump diffusions at all the three maturities.

We next use the negative jump diffusion model parameter fits from the annual maturity to compute the 10% RBC risk neutral probabilities where we take $x = y = .1$. We observe that the different models give relatively similar RBC probabilities after July 2007 (See figures 2 through 5). Additionally we provide average parameter values for the three models over the estimation period.

An important property of negative jump diffusions with a finite variation jump measure is that the upper tail is Gaussian and so implied volatility curves will flatten out on the right. From our examples it appears that this property also holds for some infinite variation jump measures.

The outline of the paper is as follows. In Section 2 we outline a set of economic fundamentals consistent with negative jump diffusions in the absence of Rally fears. Section 3 presents the stock price models used in the estimation. In section 4 we describe the procedure for computing *RBC* probabilities. Estimation Results are presented in section 5 while section 6 presents the computed *RBC* probabilities and a sample of the estimated implied volatility curves. Section 7 concludes.

2 Economic Fundamentals

A negative jump diffusion is a Lévy process and it is thus completely characterised by its distribution at unit time. Equivalently, option prices at a fixed maturity also give us information via Breeden and Litzenberger (1978) to the risk neutral distribution at this maturity. We are thus led to focus attention on the unit period distribution and ask what are the properties of risk neutral distributions of stock prices at, for example an annual horizon, and what kind of Lévy processes will match these distributions. In keeping with the general need to model a positive process we consider distributional models for the logarithm of the stock price. If we suppose that the year is a long enough time horizon and there are a sufficient number of independent finite variance effects affecting the log price at the annual horizon for central limit theorem effects to be dominant, then we may take the physical distribution of log prices to be Gaussian.

With these assumptions the terminal stock index price S may be written for a rate of return μ, volatility σ and initial stock index price S_0 as

$$S = S_0 \exp\left(\mu + \sigma Z - \frac{\sigma^2}{2}\right)$$

for a standard normal variate Z.

Under rational expectations, with a Lucas representative agent long the market index and utility function $U(S)$ we have that forward prices or spot prices under zero interest rates, w, of claims paying $c(S)$ are given by (Huang and Litzenberger (1988))

$$w = \frac{E\left[U'(S)c(S)\right]}{E\left[U'(S)\right]}. \tag{1}$$

These are economies that embed crash fears of participants long the market with low index outcomes being reweighted upwards as $U'(S)$ is large for low values of S. It is such considerations that may lead to an increase in implied volatilities for the lower strikes but as we have no short positions, there are no rally fears, and no real need for implied volatilities to rise on the right for large strikes. For a more detailed analysis of short positions on measure changes we refer the reader to Bakshi and Madan (2007) and Bakshi, Madan and Panayotov (2009).

We also know that the Lucas representative agent must price the stock correctly and hence we must have that

$$S_0 = \frac{E\left[U'(S)S\right]}{E\left[U'(S)\right]}. \tag{2}$$

It is instructive to consider what implied volatility curves one may get when options are priced using equation (1) in the presence of the restriction (2) for some reasonable choices of utility functions.

We know that constant relative risk aversion utility functions for log normal prices shift the mean and leave the volatility unchanged (Rubinstein(1976)) and so will not match the market implied volatility curves. Beyond constant relative risk aversion it was shown by Arrow (1965) that if the risky asset is to be a normal good with an elasticity of demand below unity then we should consider the class DARA (Decreasing Absolute Risk Aversion) and IRRA (Increasing Relative Risk Aversion). A simple candidate in this class is the utility function given by

$$-\frac{U''(S)}{U'(S)} = \frac{aS_0^{\theta-1}}{S^{\theta}},\ \theta < 1$$

where the scaling by $S_0^{\theta-1}$ is without loss of generality. The marginal utility function is then given by

$$U'(S) = \exp\left(-\frac{a}{1-\theta}\left(\frac{S}{S_0}\right)^{1-\theta}\right).$$

Analytical pricing is difficult in such a specification but one may easily price call options by simulation. For $\mu = 13.125\%$, $\sigma = 25\%$, we consider such a utility for $\theta = .5$ that yields the $DARA$ and $IRRA$ property. The parameter a was chosen at 2 to enforce equation (2). We then priced call options with the initial spot at 100 and strikes in the range 70 to 130 at 2 dollar intervals. These were then converted to Black Scholes implied volatilities displayed in Figure (1).

We see from this figure the characteristic smirk of implied volatilities and these are the characteristic implied volatilities of a negative jump diffusion with a right tail that is Gaussian. In fact more generally we may observe that forward call option prices $w(K)$ of strike K in the current context are given by

$$\begin{aligned} w(K) &= \frac{1}{A}\int_{\ln K}^{\infty} U'(e^x)(e^x - K)\frac{1}{\sigma\sqrt{2\pi}}\exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)dx \\ A &= \int_{-\infty}^{\infty} U'(e^x)\frac{1}{\sigma\sqrt{2\pi}}\exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)dx \end{aligned}$$

Differentiating the call option price twice with respect to the strike we see that the density of the stock price is $g(K)$ where

$$g(K) = \frac{1}{AK}U'(K)\frac{1}{\sigma\sqrt{2\pi}}\exp\left(-\frac{(\ln K-\mu)^2}{2\sigma^2}\right)$$

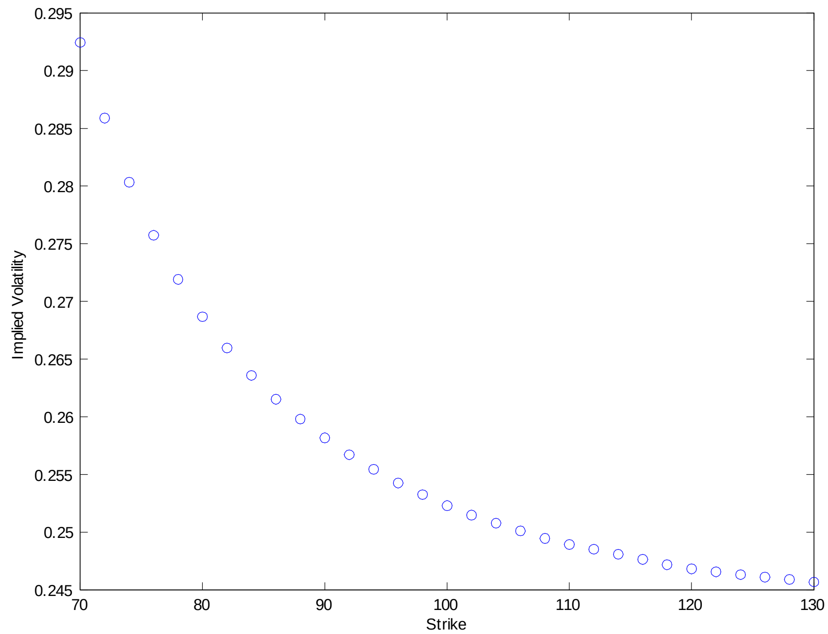

Figure 1: Implied Volatilities from a DARA and IRRA utility function

The density $f(k)$ for the log of the stock price, $k = \ln K$, is then on making the change of variable

$$f(k) = \frac{1}{A} U'\left(e^k\right) \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(k-\mu)^2}{2\sigma^2}\right)$$

and if marginal utility goes to zero as wealth goes to infinity the density is bounded above by a Gaussian density with volatility σ so implied volatilities must be bounded by σ and will decrease as we raise the strike.

The characteristic implied volatility curve of negative jump diffusion models is therefore consistent with Gaussian physical densities and the absence of short positions generating rally fears. Lévy processes with positive jumps are therefore associated with physical distributions that are fat tailed on the right or the presence of rally fears.

3 The Risk Neutral Stock Price Models

The risk neutral model for the stock price is given in terms of a pure jump Lévy process X_t with no positive jumps. The risk neutral drift is set at the interest rate r less the dividend yield q. Hence we write the stock price process S_t as

$$S_t = S_0 \exp\left((r-q)\,t\right) \exp\left(\sigma W_t - \frac{\sigma^2 t}{2}\right) \frac{\exp\left(X_t\right)}{E\left[\exp\left(X_t\right)\right]},$$

where S_0 is the initial stock price, W_t is a standard Brownian motion, σ is the volatility of the diffusion component. From this representation one sees that the stock price deflated by the forward price ($S_0 \exp((r-q)t)$) is a positive martingale composed of a geometric Brownian motion and the compensated exponential of a jump process.

The model is specified on identifying the jump measure and for the purposes of this paper we just need the characteristic exponent $\psi(u)$ given by

$$E\left[\exp\left(iuX_t\right)\right] = \exp\left(t\psi(u)\right).$$

Given the characteristic exponent $\psi(u)$ one may explicitly write down the characteristic function $\phi_t(u)$ of the logarithm of the stock price S_t at time t as

$$\phi_t(u) = E\left[\exp\left(iu\log(S_t)\right)\right] = \exp\left(t\xi(u)\right)$$

where

$$\xi(u) = iu\ln(S(0)) + iu\left(r - q - \frac{\sigma^2}{2} - \psi(-i)\right) - \frac{\sigma^2 u^2}{2} + \psi(u).$$

Standard Fourier methods of Carr and Madan (1999) may then be used to calibrate the models to option prices to estimate the risk neutral parameters. We now outline the characteristic functions associated with the three models used in this paper in three subsections.

3.1 The CGY model

The Lévy measure for the process X_t is just that of the negative side of the $CGMY$ of Carr, Geman, Madan and Yor (2002) and is given by

$$k(x) = C\frac{(-G|x|)}{|x|^{1+Y}},\ x < 0$$

and one may evaluate that in this case

$$\psi_{CGY}(u) = C\Gamma(-Y)\left((G+iu)^Y - G^Y\right)$$

We have infinite activity with $Y \geq 0$ and infinite variation with $Y \geq 1$. The number of parameters in the risk neutral model is 4, and the parameters are σ, C, G, and Y.

3.2 The model KR

Kyprianou and Rivero (2007) determine Lévy processes that they call the parent process from a special Bernstein function that is the negative of the Laplace exponent of the descending ladder heights process at unit time. They propose in their example 2 the following function for ψ.

$$\psi_{KR}(u) = -\frac{c\beta u^2 \Gamma\left(\nu + iu\beta\right)}{\Gamma(\nu + iu\beta + \lambda)}$$

for parameters $c, \beta > 0$, $\nu \geq 0$ and $\lambda \in (0, 1)$. The Lévy measure is explicitly identified in the paper and reflects infinite variation. There are five parameters in the model, σ, c, β, ν and λ

3.3 The model KRC

Kyprianou and Rivero (2007) also show that the following characteristic exponent is associated with the Bernstein function conjugate to the one used in KR and is a characteristic exponent of a pure jump Lévy process with no positive jumps.

$$\psi_{KRC}(u) = \frac{iu\Gamma(\nu + iu\beta + \lambda)}{c\beta\Gamma(\nu + iu\beta)}.$$

There are five parameters in the model, σ, c, β, ν, and λ.

4 Rally Before Crash Certificates

For any negative jump diffusion process there is an explicit solution to what is called the two sided exit problem. This problem is the determination of the probability that starting at 0 the first exit of the process from the interval $[-x, y]$ for $x, y > 0$ occurs at the upper boundary y. When working with the log return $\ln\left(\frac{S_t}{S_0}\right)$ for our risk neutral Lévy process this is precisely the risk neutral probability of a Rally of $y\%$ occurring before a Crash of $x\%$ occurs. We therefore call this value the $yxRBC$ probability that we may infer from the surface of option prices, after the calibration of a negative jump diffusion model at some maturity.

The probability is simply expressed (Bertoin (1996, page 194, Theorem 8)) in terms of what is called the scale function $W(x)$ of the Lévy process, and is given by

$$yxRBC = \frac{W(x)}{W(x+y)}. \tag{3}$$

In fact it was the search for closed forms for such scale functions that led Kyprianou and Rivero (2002) to formulate their new Lévy processes, some of which we employ here.

The scale function is also easily accessed as its Laplace transform is known in terms of the characteristic exponent (Bertoin (1996)) and we have that

$$\int_0^\infty e^{-\lambda x} W(x) dx = \frac{1}{\xi(-i\lambda) - \lambda \ln(S(0))}.$$

From our calibrated characteristic exponents of negative jump diffusions we may employ the efficient Laplace transform inversion algorithms by Abate and Whitt (1995) for example to compute the scale functions. We then get the RBC probabilities from equation (3).

5 Data and Estimation Results

We obtained data on option prices on the following seven indices, SPX, $FTSE$, $EUROSTOXX$, $N225$, $GDAXI$, HSI, and the $IBEX$ once a month for twelve days in 2007 for all the indices excepting $N225$ for which our data is for the year 2005. We used for the calibration of the three models maturities closest to a quarter, a half year and a year and strikes within 30% of the spot price. We have 7 indices, 12 days, 3 maturities and 3 models and this resulted in 756 estimations. The average number of strikes at each maturity was 35.

With a view to first assessing the ability of a negative jump diffusion to fit these option prices we constructed for all indices, days and maturities the minimum average percentage pricing error across the strikes over the three models. This gave us a vector of 252 minimum average pricing errors across the models. The average pricing error is the average absolute pricing error across the strikes divided by the average option price. Recognizing that the variance of average pricing errors may vary systmatically across maturities, for each of the three maturities we regressed the average pricing errors across the days and indices on seven dummy variables, one for each index to estimate the behavior of the best model across the indices. We report in Table 1 the coefficients of these three regressions along with the $t - statistics$ in parentheses and R squares.

TABLE 1
Results of Average Percentage
Error Regressions on Index Dummies
t-statistics in parentheses

	Maturity		
Index	3 Months	Half Year	One Year
SPX	0.0107	0.0057	0.0041
	(1.0806)	(0.8689)	(0.7611)
FTSE	0.0624	0.0389	0.0422
	(6.3072)	(5.8555)	(7.8154)
EUROSTOXX	0.0093	0.0086	0.0073
	(0.9378)	(1.2898)	(1.3610)
N225	0.0206	0.0196	0.0138
	(2.4861)	(2.9470)	(2.5645)
GDAXI	0.0033	0.0014	0.0004
	(0.3318)	(0.2186)	(0.0746)
HSI	0.0455	0.0399	0.0317
	(4.6015)	(6.0121)	(5.8663)
IBEX	0.0079	0.0053	0.0049
	(0.8041)	(0.8007)	(0.9087)
R^2	29.23%	32.53%	41.00%

We observe from Table 1 that the negative jump diffusion fits option prices well for all indices with the possible exceptions of $FTSE$, $N225$, and HSI as these are the indices with the significant t-statistics. We may compare the average percentage errors with those reported in Carr, Geman, Madan and Yor (2007). We observe that the quality of fits are generally of a comparable order and in fact for many cases superior to those reported for the Sato processes in Carr, Geman, Madan and Yor (2007). The fits are also generally better as we increase the maturity. These results with respect to maturity are consistent with the general expectation that the extent of shorting decreases with the maturity. We therefore comment further on the results for the one year maturity alone. We present in Tables 2 through 4 the average parameter values for the three models at the one year maturity.

TABLE 2
Average Parameter Values
for CGY

Index	Parameter σ	C	G	Y
SPX	0.0603	0.8095	5.2127	0.0440
FTSE	0.0810	0.7394	4.4519	0.0241
EUROSTOXX	0.0912	0.9573	4.9742	0.00004
N225	0.1236	1.2117	8.8641	0.1056
GDAXI	0.0863	0.7162	4.4218	0.0826
HSI	0.0382	5.4288	10.0935	0.0126
IBEX	0.0817	0.6699	4.0453	0.1764

TABLE 3
Average Parameter Values
for KR

Index	Parameter σ	c	β	ν	λ
SPX	0.0306	1.2154	0.3088	0.9133	0.7401
FTSE	0.0412	2.6608	0.1717	0.0013	0.8860
EUROSTOXX	0.0692	1.5652	0.1865	0.2187	0.9619
N225	0.1203	0.3783	0.2024	0.1665	0.2376
GDAXI	0.0306	0.4207	0.1955	0.0397	0.7340
HSI	0.0440	0.8228	0.2969	3.4002	0.9315
IBEX	0.0573	0.4527	0.1557	0.4203	0.7427

TABLE 4
Average Parameter Values
for KRC

Index	Parameter σ	c	β	ν	λ
SPX	0.0540	1.4496	0.2009	0.8388	0.00179
FTSE	0.0667	1.3612	0.2532	0.6599	0.00045
EUROSTOXX	0.0992	1.1126	0.0117	0.0893	0.00015
N225	0.1359	1.8854	1.1861	3.1894	0.00822
GDAXI	0.0721	1.4662	0.1444	0.6718	0.00852
HSI	0.0386	1.0872	0.0252	0.0797	0.00022
IBEX	0.0761	1.7497	0.2880	1.1969	0.00083

We observe from these tables that $N225$ and HSI have parameter values that contrast with the other indices. The diffusion coefficients are comparable across models and indices excepting $N225$ where it is substantially higher for all the three models. However, we note that the $N225$ data is for a different period, two years prior to the other indices. For the one year maturity the best model

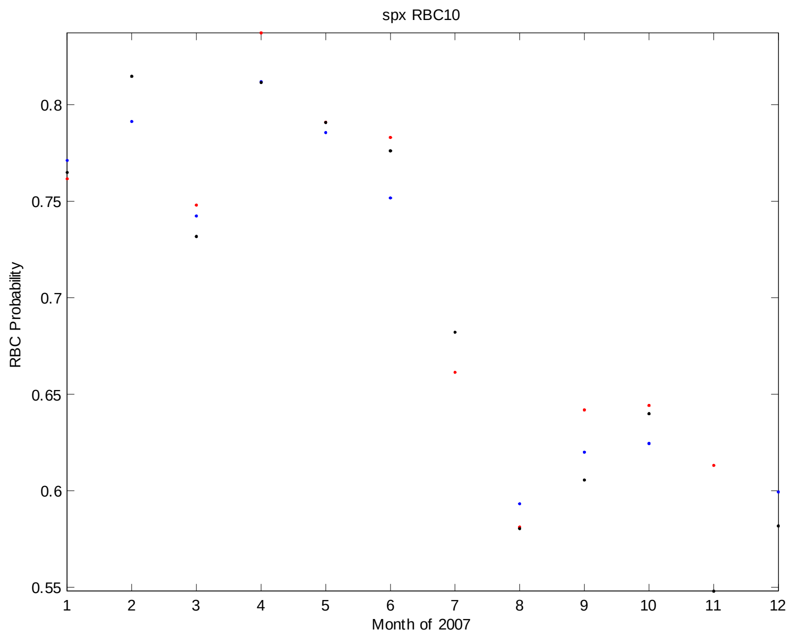

Figure 2: Rally Before Crash Probabilities for SPX over 2007 using CGY, KR, and KRC

out of the 84 cases was KRC in 51 cases followed by KR in 19 cases and CGY in 14 cases. However the differences between models in the average percentage error is not very large and the average difference in the average percentage error between the best and second best is only 24 basis points.

6 Rally Before Crash Results

For each index we used the three models with the parameter values as fitted at the one year maturity and computed via inverse Laplace transforms the scale functions and then the risk neutral probability of a 10% rally before a 10% crash. The results are best presented graphically and we present as a sample just the results for the SPX, $FTSE$, $EUROSTOXX$ and $GDAXI$. The different colors are for the different models with blue for CGY, red for KR and black for KRC. We notice that the models are closer to each other earlier in the year and a little further apart later in the year. The risk neutral probability of a 10% percent rally before a 10% crash fell uniformly across the models and indices by approximately 10 points post July 2007 as the subprime crisis unfolded. A model was used only if the average pricing error was below the generous cutoff of 15% and there were more than 5 options available for the calibration.

For negative jump diffusions with a finite variation jump measure the right

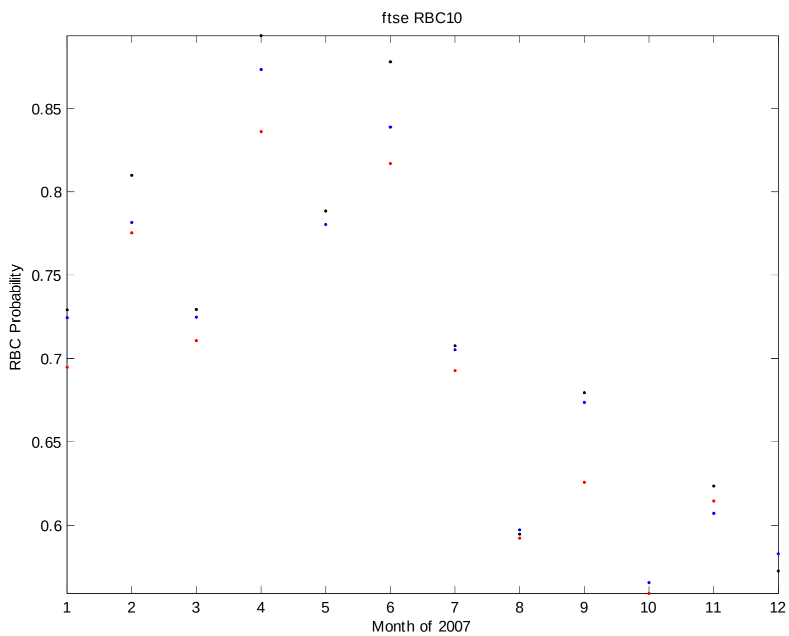

Figure 3: Rally Before Crash Probabilities for FTSE over 2007 using CGY, KR and KRC.

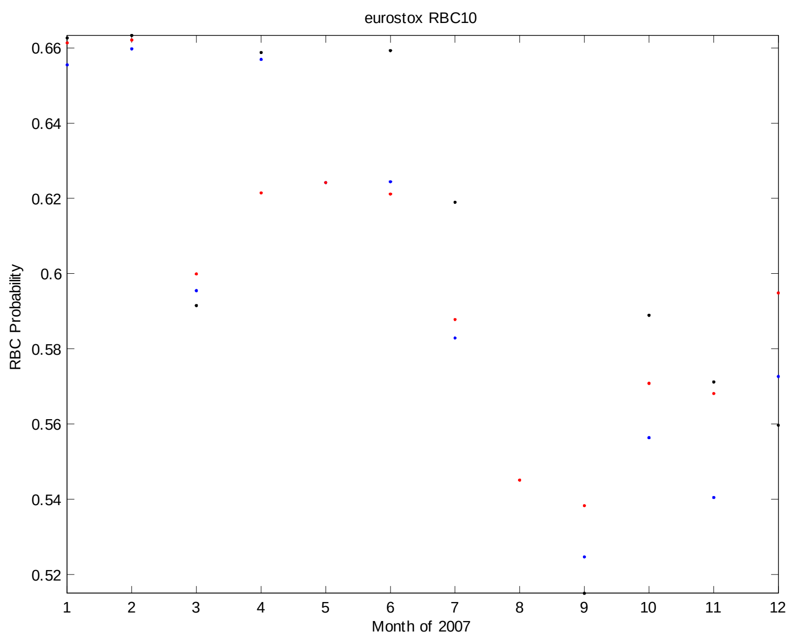

Figure 4: Rally Before Crash Probabilities for EUROSTOXX over 2007 using CGY, KR and KRC.

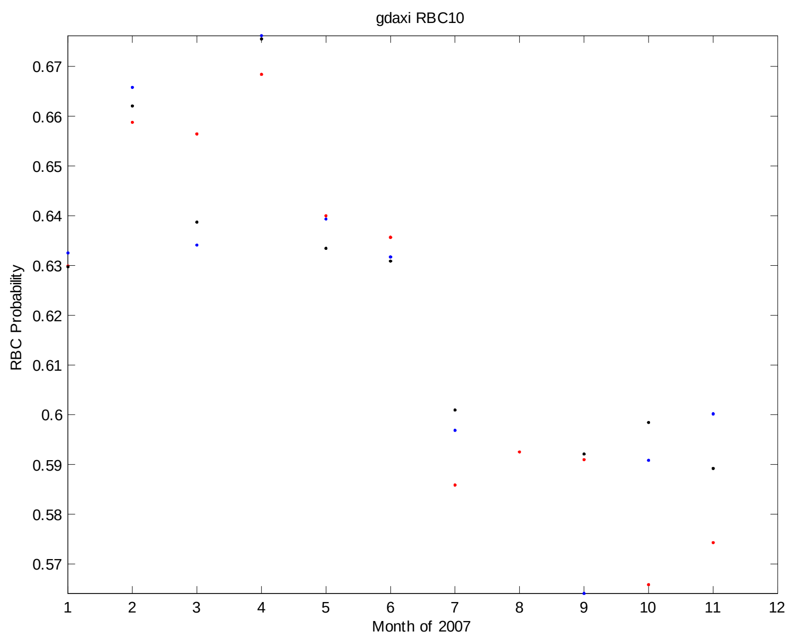

Figure 5: Rally Before Crash Probabilities for GDAXI using CGY, KR and KRC

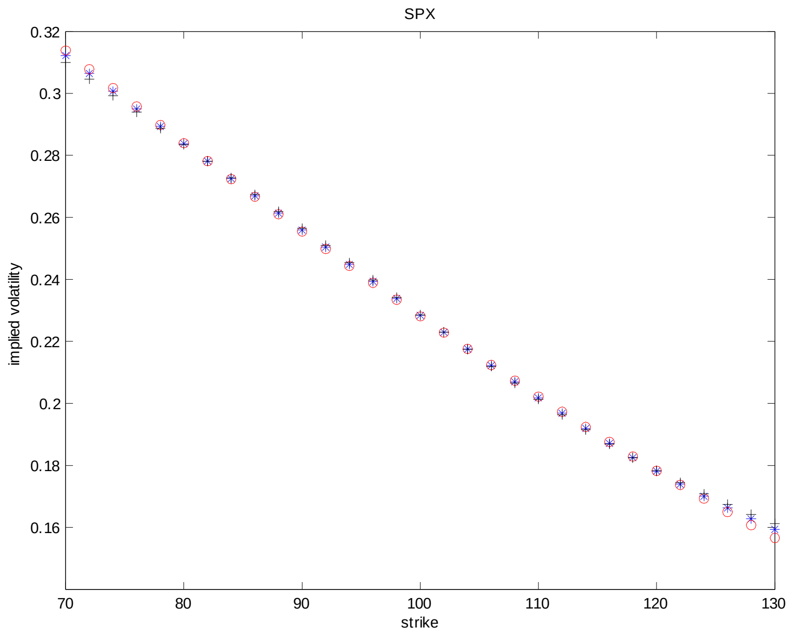

Figure 6: SPX implied volatility curves for CGY, KR, KRC on December 14 2007.

tail of the probability density of log prices at unit time is Gaussian and implied volatilities will converge to the level reflected in the diffusion component. We believe that this pattern is also maintained for the infinite variation models. We present the implied volatility curves for the last day of the estimation in 2007 for all three models for the SPX, $FTSE$, $EUROSTOXX$ and the $GDAXI$. We observe that the curves are very close to each other and display the characteristic smirk of a negative jump diffusion. The model CGY is represented by a blue star, for KR we use a red circle while for KRC we employ a black plus sign.

7 Conclusion

We investigate the pricing of index options in world index options markets using a risk neutral model for the logarithm of the stock price as a Lévy process with a diffusion component and no positive jumps. The asymmetry is motivated economically by the possible absence of significant short positions, especially at the longer maturities and this feature makes the financial markets asymmetric with respect to long and short positions. Additionally we have the impact of call overwriting strategies that have a similar effect. Mathematically we are encouraged to use such models as for such processes we may compute interesting probabilities like the probability of a $y\%$ rally occuring before an $x\%$ crash.

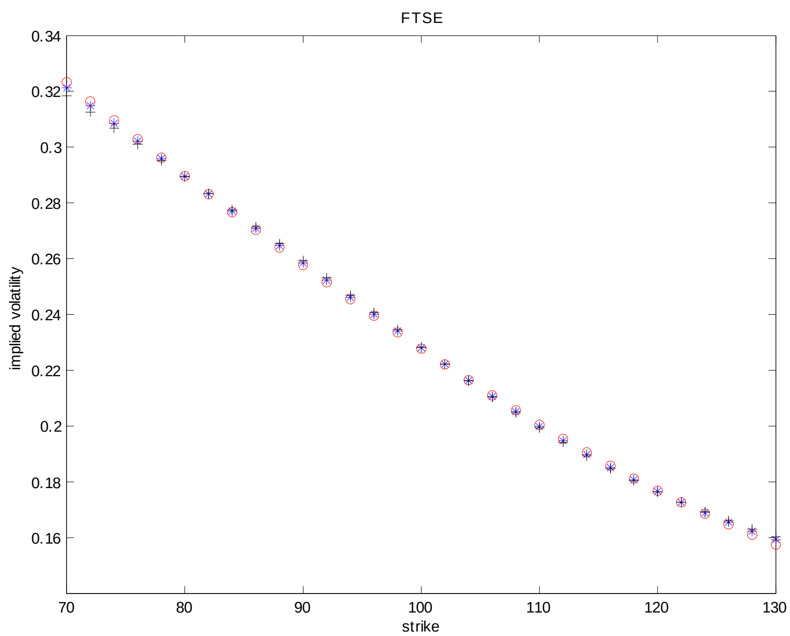

Figure 7: FTSE implied volatility curves for CGY, KR, KRC on December 14 2007.

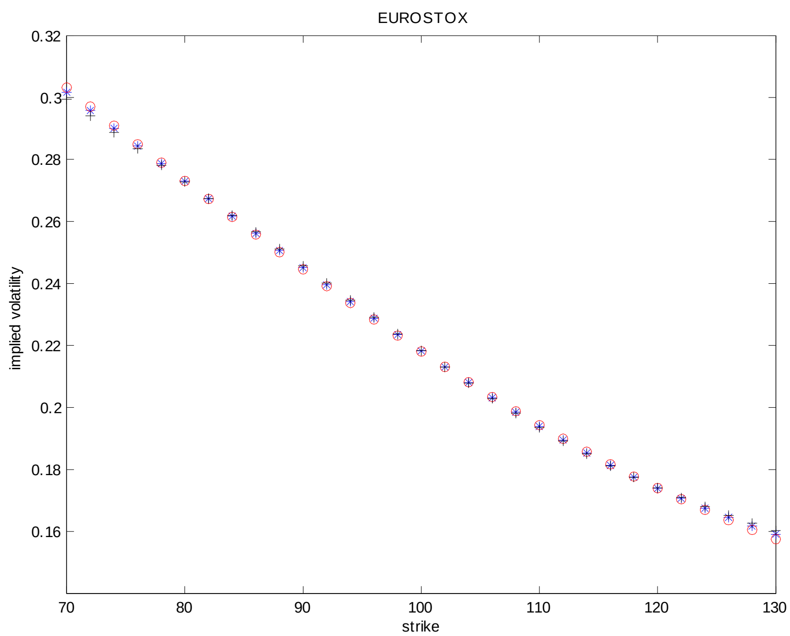

Figure 8: EUROSTOXX implied volatility curves for CGY, KR, KRC on December 14 2007.

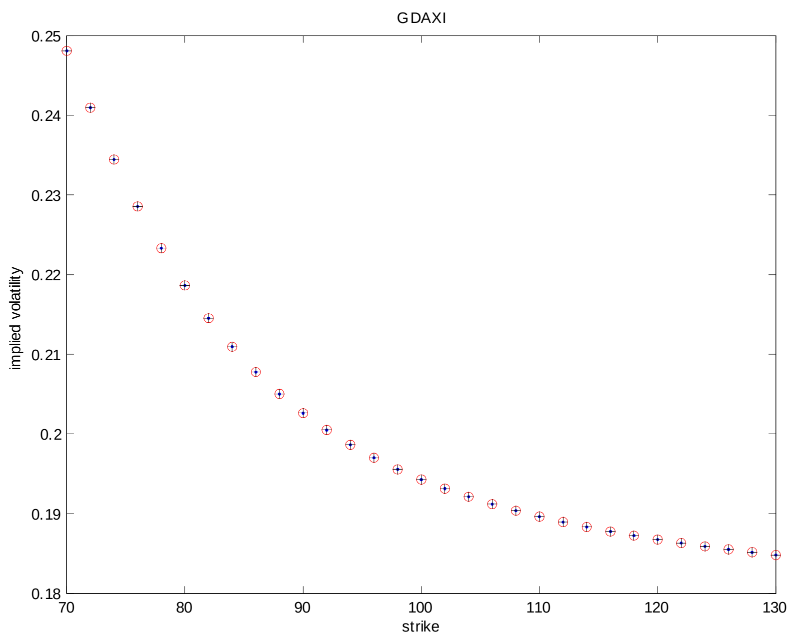

Figure 9: GDAXI implied volatility curves for CGY, KR, KRC on December 14 2007.

Finite maturity versions of these probabilities trade in foreign exchange markets as digital touch/no touch securities.

Three models are used in our investigations, the $CGMY$ model with no positive jumps that effectively sets M to infinity and we call it here CGY. In addition we employ two recent variations of CGY proposed by Kyprianou and Rivero (2007) that we here call KR and its conjugate model KRC.

We show that all three models perform relatively well in pricing index options for the indices SPX, $FTSE$, $EUROSTOXX$, $N225$, $GDAXI$, HSI and $IBEX$. The performance on the $FTSE$, $N225$, and HSI is significantly inferior. This leads us to conjecture that these markets may be exposed to the presence of a fear of rallies possibly caused by short positions on the part of highly risk averse participants. The resulting demand for upside calls can lift the right tail of the stock price density above a Gaussian density creating a need for a model with positive jumps.

We also compute the risk neutral probabilities of a 10% rally before a 10% crash and show that this probability fell by 10 points after July 2007 as compared to the start of the year. Finally we present the characteristic or signature implied volatility curve of a negative jump diffusion that generates a Gaussian tail on the right with implied volatility curves that flatten out on the right at the level of the diffusion component of the model.

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