Hedge Fund Performance: Sources and Measures

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Abstract

The concept of the gamma of a financed return as the highest level of stress that a return distribution can withstand is introduced. Stress is measured by positive expectation under a concave distortion of the return distribution accessed. Four distortions introduced in Cherny and Madan (2008) are employed in studying the distribution of returns available in the hedge fund universe. It is shown that the skewness, peakedness and tailweightedness of the standardized investment return significantly affects the Sharpe ratios required to reach a target gamma level.

1   Introduction

Hedge funds employ a wide array of dynamic, event driven, and relative value trading strategies to access statistical arbitrages in financial markets. Consequently one expects non-Gaussian return distributions (Brooks and Kat (2002), Agarwal and Naik (2004), Malkiel and Saha (2005)). Though the Sharpe ratio adequately evaluates performance for Gaussian returns, its limitations in non-Gaussian contexts are well recognized (Bernardo and Ledoit (2000), and Kat (2003)).

Recently, Cherny and Madan (2008) have proposed a family of new performance measures improving on the economic properties of the Sharpe ratio and the Gain-loss ratio, proposed in Bernardo and Ledoit (2000). These new measures are particularly suited to non-Gaussian environments, by taking account of the entire distribution of returns in evaluating performance.

In this paper we apply these new performance measures to the hedge fund universe and thereby describe the sources of hedge fund return performance. We first establish that performance targets in terms of the new measures imply an analytically determined required Sharpe ratio. The input for determining the required Sharpe ratio is the standardized zero mean unit variance return

distribution. It is observed that among the higher moments of this standardized return distribution negative skewness decreases performance reflected in an associated increase in the required Sharpe ratio. Furthermore, we discover that kurtosis is better decomposed into its components of peakedness and tailweightedness, both of which contribute positively towards kurtosis but have opposite effects on performance and required Sharpe ratios.

The new performance measures introduced in Cherny and Madan (2008) in conjunction with the results of this paper offer an axiomatically founded solution to the problem of adjusting Sharpe ratios for non-Gaussianity. Many other attempts have also been made in this direction in the literature and include as examples Sortino and Price (1994), Stutzer (2000), Dowd (2000), Keating and Shadwick (2002), Kaplan and Knowles (2004), and Ziemba (2005). For attempts more closely linked to the theoretical foundations of expected utility theory we cite Hodges (1998), Madan and McPhail (2000) and Koekebakker and Zakamouline (2007). Importantly, we note, however, that Cherny and Madan (2008) show that the acceptance sets based on expected exponential utility (or the Tilt coefficient in Cherny and Madan (2008)) fail to satisfy the axioms introduced for performance measures therein. In particular the acceptance sets fail to be convex sets, they are also not scale invariant and are not consistent with second order stochastic dominance. The solution offered here meets all these requirements by design.

Though theoretically one may consider the new measures in a portfolio context on asking whether the addition of a fund to an existing portfolio improves performance. In this first application of the new measures, however, we work in the original context of Sharpe ratios and Gain-Loss ratios by using the measures in isolation. We therefore leave for future research the extension to a portfolio context. The new measures could also be of interest to discussions of hedge fund replication as they describe attributes of hedge fund returns differentiated from the more prevalent statistics of alphas and betas. In keeping with this terminology we refer to the new measures as the gammas of the hedge fund return outcomes.

The new measures aim at measuring the level of acceptability of a potentially risky outcome. This level is by design infinite for all arbitrages. More generally one associates with each level of acceptability, or gamma level, γ for short, a convex cone of random variables acceptable at this level. It follows that linear combinations and positive scalar multiples of variables acceptable at level γ are also acceptable at the same level. It is important to note that the acceptability level is not a utility or preference ordering as all arbitrages have the single acceptability level of infinity and furthermore acceptability levels do not differentiate between scaled outcomes.

Intuitively speaking, high gamma levels are to be attained when the return distribution withstands a high level of stressed sampling. A simple example illustrates such a procedure. Suppose the return distribution is such that the expectation of the minimum of n independent draws from the distribution is still positive, then we say that it is acceptable at level n. The gamma level of the return distribution is then the largest number γ such that the expectation of

the minimum of $1 + \gamma$ draws from the distribution is still positive. Formally one constructs such an expectation for a random variable with distribution function $F(x)$ by merely sampling from the distribution function $\Psi_\gamma(F(x))$ where

$$\Psi_\gamma(y) = 1 - (1 - y)^{1+\gamma} \tag{1}$$

The composition of the two distribution functions $F(x)$ and $\Psi_\gamma(x)$ is called a concave distortion of the original distribution function. This particular concave distortion is termed $MINVAR$ by Cherny and Madan (2008) as it is focused on minimum outcomes. Convex distortions applied to the complementary distribution function of utility have recently been considered by Jin and Zhou (2007) in the context of modeling objectives consistent behavioral finance. We apply a sequence of concave distortions to the distribution function instead and seek the highest stress level for the distortion as the γ level of the strategy.

Alternatively one may define other patterns of stress by choosing other distortion functions Ψ_γ, noting that these functions must be for each γ a concave distribution function on the unit interval that increases pointwise in γ to unity as γ tends to infinity. The paper employs three other distortions.

A simple computation shows that these stressed expectations are expectations under a measure change given by $\Psi'_\gamma(F(x))$. One therefore sees that low outcomes with $y = F(x)$ near zero receive a higher weight while simultaneously good outcomes for y near unity receive a lower weight. Two of the distortions used have the economically attractive property of a reweighting scheme that goes to infinity for large losses and to zero for large gains. Our study confirms that these improved economic properties lead to the recommendation that required Sharpe ratios based on these distortions better reflect the adjustments needed for exposure to skewness, peakedness and tailweightedness in return outcomes.

We note that there is an extensive literature dealing with the asset pricing implications of higher moments beginning with Kraus and Litzenberger (1976) with more recent contributions by Bansal, Hsieh and Viswanathan (1993), Bansal and Viswanathan(1993), Chapman (1997), Harvey and Siddique (2000), and Dittmar (2002). Much of this work is concerned with higher moment effects in systematic factors and their implications for the prices of liquid assets. The problem we address here is fundamentally different. We are concerned with the effects of higher moments of the full distribution on the gamma or acceptability level of the cash flow or strategy.

A word of warning is also called for with respect to strategies aimed at maximizing Sharpe ratios as studied for example in Goetzmann, Ingersoll, Spiegel and Welch (2002). Depending on how a Sharpe ratio is increased, it may increase or decrease the gamma of a cash flow. The results of this paper may be employed to shed further light on whether one is in fact moving closer to or further from an arbitrage in adopting a particular strategy to raise the Sharpe ratio. In fact the findings of Agarwal and Naik (2004) with respect to accessing left tail risk could be refined to determine whether acceptability or gamma is increased by particular hedge fund strategies. The method of determining gamma could also be applied to other popularly studied strategies like pairs trading (Gatev, Goetzmann and Rouwenhorst (2006)) among others.

The outline of the paper is as follows. Section 2 establishes the tradeoff between centered and scaled distributions being accessed and the Sharpe ratios required to attain a particular γ. The methods are implemented in Section 3 for a universe of 527 hedge fund returns with a five year history of monthly returns permitting estimation by maximum likelihood of the return distribution F in the $CGMY$ class of densities (Carr, Geman, Madan and Yor (2002)). In Section 4 we derive the required Sharpe ratios for all 527 hedge funds in accordance with the theory of Section 2, using 4 different concave distortions to stress cash flows to particular gamma levels. Section 5 reports on the summary relationship between required Sharpe ratios and measures of skewness and kurtosis, and kurtosis partitioned into peakedness and tailweightedness. Section 6 concludes.

2 The gamma of a financed return

We develop in this section the theory for inferring the gamma of an investment return from the level of the Sharpe ratio and the distribution function of the return standardized to zero mean and unit variance. We note that these return distributions come from strategies combining long and short positions and so we take them to be potentially unbounded in both directions. Hence the return being accessed is given by the real valued random variable X, with a distribution function $F_X(x)$. The acceptability level and hence the gamma of the return will be determined completely by its distribution function.

We index by the positive real $\gamma \geq 0$ the level of acceptability of the cash flow with arbitrages being acceptable at level infinity. We follow Cherny and Madan (2008) and introduce a family of concave distortions given by concave distribution functions Ψ_γ on the unit interval beginning with $\Psi_0(a) = a$ and increasing the degree of concavity as we increase γ.

In this formulation, the random variable X is acceptable at level γ provided

$$\int_{-\infty}^{\infty} x d\left(\Psi_\gamma(F_X(x))\right) \geq 0. \tag{2}$$

The fact that the benchmark level on the right hand side of (2) is zero is a consequence of requiring acceptable cash flows to contain all arbitrages and hence all positive random variables however small their expectation. The first example of a useful family of concave distortions was noted in the introduction as $MINVAR$. The acceptability condition (2) here requires that the expectation of the minimum of $(1 + \gamma)$ independent draws from the distribution of X be positive. This acceptability condition tests for positive worst case expectations by constructing the worst case using the minimum of independent draws.

Another way of taking a worst case called $MAXVAR$ in Cherny and Madan (2008) is to obtain the distribution G of such a random variable that after drawing from G, independently $(1+\gamma)$ times and taking the maximum outcome we get the distribution F_X. In this case

$$G(x) = F_X(x)^{\frac{1}{1+\gamma}}$$

and

$$\Psi_\gamma(y) = y^{(1+\gamma)^{-1}}, \ 0 \le y \le 1 \tag{3}$$

We may then combine these two approaches to form the concave distortions $MAXMINVAR$ given by

$$\Psi_\gamma(y) = \left(1 - (1-y)^{(1+\gamma)}\right)^{\frac{1}{1+\gamma}}$$

where we first take the minimum of independent draws and then find the distribution from which to draw, take the maximum outcome to get the law of the minimum of $(\gamma + 1)$ draws.

The reverse procedure yields $MINMAXVAR$ with

$$\Psi_\gamma(y) = 1 - \left(1 - y^{\frac{1}{1+\gamma}}\right)^{1+\gamma}.$$

Cherny and Madan (2008) note that the density of the latter two approaches with respect to the law of X tends to infinity and zero as x tends to minus and plus infinity, respectively. This property is in accordance with measure changes seen in economics from a marginal utility perspective where large losses are exaggerated towards infinity while large gains are deflated towards zero. For $MINVAR$ the density is bounded by $\gamma + 1$ on the left while for $MAXVAR$ it approaches $(1/1 + \gamma)$ on the right.

We may define acceptability using any of these candidate distortions to stress the law of X before evaluating an expectation. We now construct the standardized (i.e. centered and scaled) distribution being accessed by the random variable X. Let μ be the mean of X and σ its standard deviation and define the standardized random variable Z such that

$$X = \mu + \sigma Z \tag{4}$$

The distribution of Z by virtue of a zero mean and unit variance accesses higher moments and under Gaussianity it would be the standard normal variable. In non-Gaussian domains the density of Z targets, by design, exposure to different levels of skewness and kurtosis among other aspects of the distribution.

Consider now the expectation under a concave distortion for the random variable Z with distribution $F_Z(z)$. By the concavity of Ψ we know that $\Psi'(F_Z(z))$ in (2) overweights negative outcomes relative to positive ones and so reduces the expectation under the distortion below the undistorted value of 0. Let $c_Z(u)$ be the negative of the stressed expectation of Z under the distortion Ψ_γ or

$$c_Z(\gamma) = - \int_{-\infty}^{\infty} z d \left(\Psi_\gamma \left(F_Z(z) \right) \right).$$

Theorem 1 *The random variable X attains the acceptability level γ if and only if its Sharpe ratio*

$$SR(X) = \frac{\mu}{\sigma} \ge c_Z(\gamma) \tag{5}$$

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Proof. See Appendix ∎

By Theorem 1 the function $c_Z(\gamma)$ is seen to be the Sharpe ratio needed to attain the acceptability level γ. Intuitively it is clear that if Z is negatively skewed then the expectation under a concave transformation would be more negative and hence the Sharpe ratio needed for any level γ would be higher. The effect of an increase in just the kurtosis is less clear. In fact it is well known that kurtosis is made of two effects, one associated with the peakedness of a distribution and the other with the weight in the tails. The net effect is generally ambiguous. In this connection we note that Dittmar (2002) following Darlington (1970) argues for kurtosis being dominated by tailweightedness. This is incorrect as observed in Moors (1986). We shall later in our regression summary separate out these effects.

The gamma of a zero cost cash flow or self financed strategy is determined on solving in γ the equation

$$\frac{\mu}{\sigma} = c_Z(\gamma). \tag{6}$$

Though the mean, variance and Sharpe ratio appear explicitly in equation (6) the gamma of a strategy goes way beyond a mean variance analysis. This is because the distribution of the entire centered and scaled random variable Z is synthesized in the function $c_Z(\gamma)$.

We now establish some general properties about the behavior of the gamma target Sharpe ratio as a function of γ.

Theorem 2 *The required Sharpe ratio for level γ, $c_Z(\gamma)$, is increasing in γ and provided $\frac{\partial^2}{\partial \gamma^2} \Psi_\gamma < 0$ then $\frac{\partial^2}{\partial \gamma^2} c_Z(\gamma) < 0$*

Proof. See Appendix ∎

To get some further insights into the nature of the gamma target Sharpe ratios engineered by higher moment access it is instructive to construct the function $c_Z(\gamma)$ for a class of distributions that give us access to variations in the structure of higher moments. A robust class in this regard is given by the law at unit time of a pure jump Lévy process that may be left or right skewed by having a smaller or larger rate of positive jumps compared to equally sized negative ones. A particularly manageable class in this regard is the variance gamma model of Madan and Seneta (1990), Madan, Carr and Chang (1998) or its generalization to the $CGMY$ class of Lévy processes introduced in Carr, Geman, Madan and Yor (2002). The characteristic exponent $\psi(\xi)$ of the centered and scaled to unit variance $CGMY$ random variable Z is the function

$$
\begin{aligned}
\phi(\xi) &= E\left[\exp\left(i\xi Z\right)\right] = \exp\left(\psi(\xi)\right) \\
\psi(\xi) &= -i\xi \frac{G^{Y-1} - M^{Y-1}}{(Y-1)\left(M^{Y-2} + G^{Y-2}\right)} \\
&\quad + \frac{(M - i\xi)^Y - M^Y + (G + i\xi)^Y - G^Y}{Y(Y-1)\left(M^{Y-2} + G^{Y-2}\right)}
\end{aligned}
$$

The parameter C was substituted out to organize a unit variance and

$$C = \frac{1}{\Gamma(-Y)Y(Y-1)\left(M^{Y-2} + G^{Y-2}\right)}.$$

For future reference we note that the skewness and kurtosis of the centered and scaled $CGMY$ law at unit time for this Lévy process are

$$Skewness \;\; = \;\; \frac{(2-Y)\left(M^{Y-3} - G^{Y-3}\right)}{M^{Y-2} + G^{Y-2}} \tag{7}$$

$$Kurtosis - 3 \;\; = \;\; \frac{(2-Y)(3-Y)\left(M^{Y-4} + G^{Y-4}\right)}{M^{Y-2} + G^{Y-2}} \tag{8}$$

We observe that as Y approaches the level 2 the process gets closer to Gaussian with skewness and excess kurtosis vanishing. The parameter Y controls the relative weight given to small moves as compared with large ones. The skewness is computed by evaluating $i\psi'''(0)$ while the excess kurtosis is given by $\psi^{(iv)}(0)$. Another robust class of distributions which could be investigated in this context is the class of generalized hyperbolic (GH) distributions (Eberlein (2001)).

We present in Figure 1 graphs of the function $c_Z(\gamma)$ for different values of the parameters G, M, Y and the $MINMAXVAR$ concave distortion. The graphs are obtained by inverting the characteristic function for the distribution function, passing it under the $MINMAX$ concave transformation and numerically evaluating the expectation of the stressed distribution.

We observe that negative skewness associated with low values for G and high values for M are associated with significantly higher gamma target Sharpe ratios. Hence the mere attainment of a higher Sharpe ratio does not amount to an increase in the cash flow gamma. It may even be a worse outcome if the Sharpe ratio is not sufficiently high to compensate for the additional skewness. Similarly a low Sharpe ratio may yield a high gamma if the target curve is decreased by the type of investment made.

We may see the net effect of kurtosis by setting $G = M$ to get zero skewness and varying M to reflect different kurtosis levels for $Y = .5$. We present in Figure 2 the effect of kurtosis on the required Sharpe ratio. We observe that an increase in kurtosis reduces the required Sharpe ratio for entering any particular cone of acceptability. It appears that the effect of peakedness might dominate that of tailweightedness.

3   The Gamma Levels of Hedge Funds

Perhaps the segment of the financial market most focused on accessing non-Gaussian returns by engaging in dynamic or event driven arbitrage strategies is the universe of hedge funds. As they attempt to exploit arbitrage opportunities they may in fact succeed in attaining high levels of gamma for the four acceptability indices discussed here. It is therefore interesting to ascertain the level of
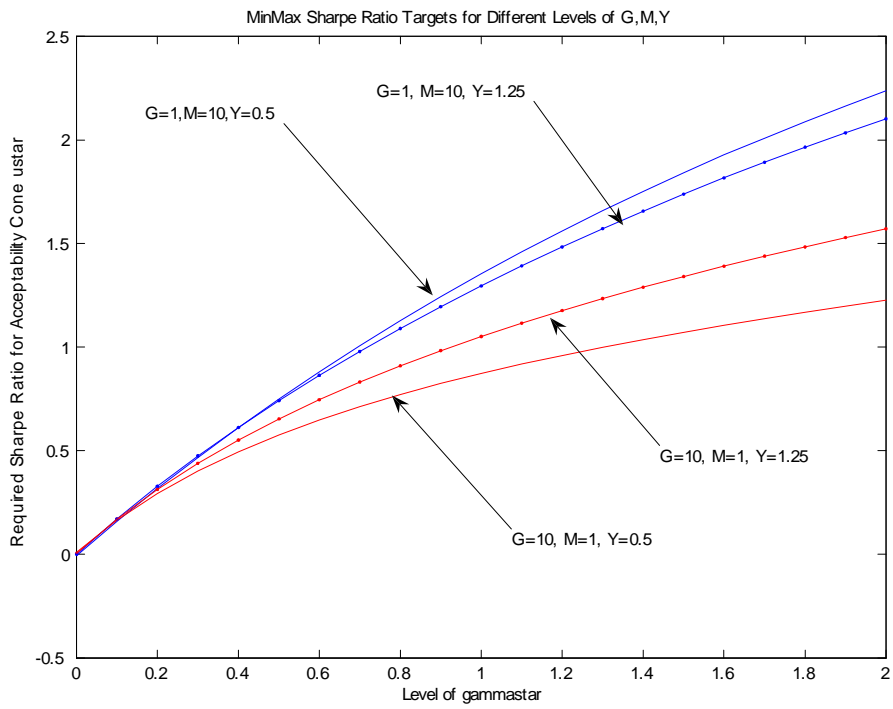
Figure 1: Sharpe Ratios Required for different acceptability levels for access to centered and scaled $CGMY$ random variables at unit time.
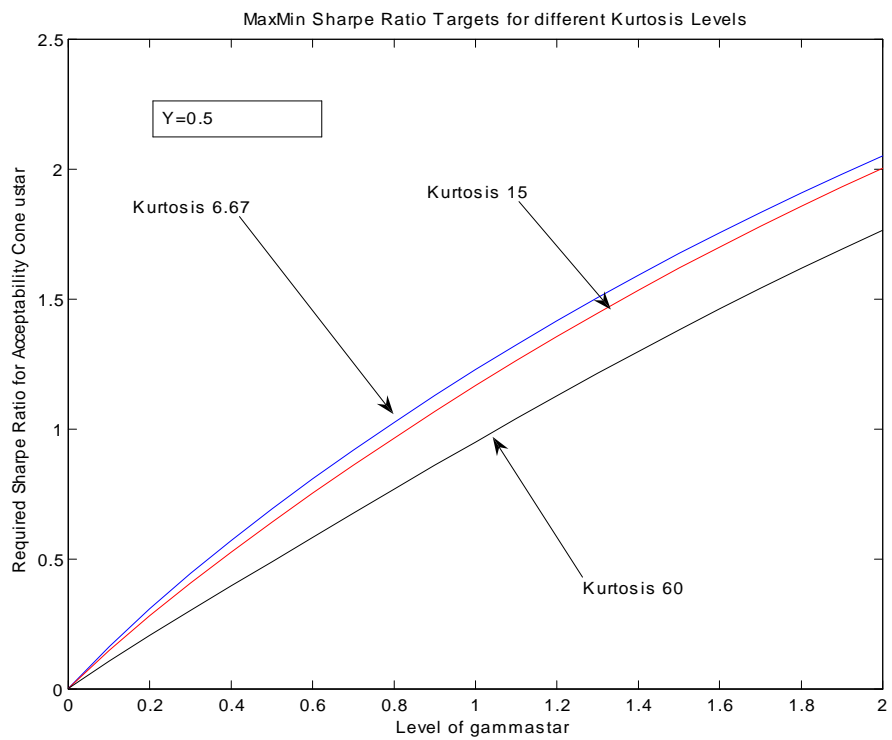
Figure 2: MAXMIN Required Sharpe Ratios for different kurtosis levels for the centered and scaled CGMY variate with zero skewness

gamma attained in the hedge fund universe. For this purpose we obtained data on some 662 hedge funds with a continuous history of five years of monthly data and a positive mean return. We recognize the limitations of this procedure as it is likely to bias upwards the performance levels both on account of the survival bias and on account of the long reporting period with the inherent practice of smoothing such data by fund managers. The general qualitative results on the relationship of performance to other statistics of the data may yet be robust to these limitations. By way of a comparative backdrop we also extracted data on 44 single name stocks and 10 market indices for a similar five year period using for this purpose 60 monthly nonoverlapping returns. Monthly returns were used for the stocks and the indices to keep the analysis of the stocks and indices on the same basis as the hedge funds. We thus had 716 sets of 60 monthly return observations.

For each of these return series we computed the level of volatility, skewness and kurtosis. We report in Table 1 the volatilities, skewness, and kurtosis in the single stocks, the indices and the hedge funds at each of seven quantile levels. We observe considerable differences in line with stylized observations on hedge fund returns. They have substantially lower volatilities but yet couple this with much higher levels of skewness and kurtosis. For example at the $95th$ percentile the skew is as high as 1.7754 while at the 5% level it is -2.0812. The corresponding numbers for single stocks and indices are .5612, $-.0695$ and $-.8751, -0.9104$. The single stock kurtosis levels at the 95% level are 6.9330 with indices at 4.7377 while the hedge funds have 17.7058. Yet this picture on skews and kurtosis is associated with much lower volatilities across the board. Again at the 95% level the hedge funds have a volatility of 4.39% with the indices at 6.48% and single name stocks at 11%. Taken at face value these observations are consistent with substantial differences between how hedge funds earn their monies and what one would get from investing in stocks or indices. One may expect that the levels of the gamma attained by hedge funds are possibly also higher. Some part of these effects are related to the survival and smoothing

biases already noted.

TABLE 1
quantiles for vol, skew and kurtosis

		quantiles						
		.01	.05	.25	.5	.75	.95	.99
	Stocks	.0361	.0448	.0613	.0713	.0895	.1100	.1431
Vol	Indices	.0343	.0343	.0384	.0538	.0597	.0648	.0648
	Funds	.0054	.0069	.0101	.0152	.0243	.0439	.0590
	Stocks	-1.0297	-.8751	-.4131	-.1327	.0612	.5612	1.3663
Skew	Indices	-.9104	-.9104	-.8226	-.4891	-.2740	-.0695	-.0695
	Funds	-4.6922	-2.0812	-.5355	-.0670	.4863	1.7754	2.9625
	Stocks	2.3771	2.4691	2.8231	3.4607	4.2637	6.9330	10.4426
Kurtosis	Indices	2.5240	2.5240	2.7213	3.7157	4.5908	4.7377	4.7377
	Funds	2.4203	2.7696	3.6528	4.8035	7.5710	17.7058	37.9672

To evaluate the gamma levels one needs to access the distribution function, distort it by making a worst case expectation computation using concave distribution transformations and then evaluate the highest level γ of the concave transformation for which the expectation is positive. With 60 data points the construction of a nonparametric distribution function is on shaky grounds. We therefore recommend the use of a robust parametric model and we suggest that one minimizes the number of parameters to be estimated by a nonlinear optimizer like maximum likelihood estimation.

We further recognize that the return distribution being estimated is the result of potentially many trades conducted in varied markets over the month. Hence the monthly return aggregates the result of many possibly independent outcomes. This suggests that a limit law like the Gaussian distribution may be relevant. However, the extensive non-Gaussian nature of these distributions argue against the Gaussian model. Hence we are led to other limit laws. The collection of all limit laws were characterized by Lévy (1937) and Khintchine (1938). These are the class of self decomposable random variables and they contain many non-Gaussian alternatives. Self decomposable random variables are infinitely divisible and therefore are a subclass of the distributions of Lévy processes at unit time. However, to be a limit law the arrival rate of jumps as described by the Lévy density must have a special structure. First the process must have infinite activity as discussed for example in Carr, Geman, Madan and Yor (2002). Secondly, when we multiply the Lévy density $k(x)$ by $|x|$ we must get a function that is decreasing in x for positive x and increasing in x for negative x (see for example Sato (1999)). A classic example of such a process is the variance gamma model of Madan and Seneta (1990) and Madan, Carr and Chang (1998) where the required function that is increasing and decreasing for negative and positive x respectively is just $\exp(-|x|)$. We employ the further generalization attained in the $CGMY$ model of Carr, Geman, Madan and Yor (2002).

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The centered and scaled $CGMY$ class considered in the previous section are a flexible collection of such limit laws for all values of the parameters. We therefore expect monthly hedge fund returns to be described well by such distributions. The distributions have been tested on daily stock returns and can in any case be further tested for any particular application. Here we illustrate the calculations using this family of densities. We expect only small differences from movements in Y that really controls the level of small activity. With a view to minimizing parameters we centered and scaled the return data, fixed Y at 0.5 and estimated just G, M for 716 sets of the return data.

The levels of gamma for the hedge funds were then generated by constructing simulated cash flows of 1000 readings by drawing from the centered and scaled estimated distribution by the inverse distribution function method applied to a draw of uniform random numbers. The distribution function was obtained by Fourier inversion techniques. Simulated cash flows were obtained on scaling by the known standard deviation and adding back the mean. This method would give us a matrix of 1000 readings of cash flows on 716 underliers. However, we dropped underliers that were estimated close to Gaussian distributions and those with negative means as our interest is in the structural relationship of non-Gaussianity to the gamma level of acceptability that is positive only with a positive mean. We were left with 27 stocks, all the 10 indices and 527 hedge funds. There was then a total of 564 underliers.

For these 564 cash flows with 1000 simulated readings we computed the gamma on building an empirical distribution function out of the cash flow, distorting it by level γ, evaluating the expectation of the distorted distribution function by numerical integration and maximizing the level of the index γ yielding a positive distorted expectation. All the four new performance measures of Cherny and Madan (2008) based on the four distortions of $MINVAR$,

$MAXVAR$, $MINMAXVAR$ and $MAXMINVAR$ were used.

TABLE 2
Gamma Levels of Funds From Standardized CGMY

Based on		quantiles						
		.01	.05	.25	.5	.75	.95	.99
	Stocks	0	0	.0205	.1177	.2444	.3243	.3525
MINVAR	Indices	0	0	.0515	.1265	.1945	.2492	.2492
	Funds	.0438	.1640	.4216	.7175	1.1345	2.0668	3.43
	Stocks	0	0	.0188	.0883	.1983	.2779	.3097
MAXVAR	Indices	0	0	.0423	.0957	.1564	.1964	.1964
	Funds	.0407	.1374	.3309	.4966	.7142	1.2346	1.9026
	Stocks	0	0	.0098	.0494	.1079	.1426	.1563
MAXMINVAR	Indices	0	0	.0231	.0535	.0863	.1062	.1062
	Funds	.0214	.0738	.1760	.2679	.3892	.6591	.9670
	Stocks	0	0	.0098	.0488	.1049	.1372	.1499
MINMAXVAR	Indices	0	0	.0229	.0527	.0844	.1032	.1032
	Funds	.0212	.0726	.1673	.2495	.3529	.5645	.7887

The results are presented in Table 2. We observe that subject to the survival and smoothing biases the hedge funds do indeed access much higher levels of gamma. The highest values are for those based on $MINVAR$ that has the least discount on large losses. This is followed by $MAXVAR$ and then we have dual worst case constructions with $MINMAXVAR$ being the severest of the four. However, we do have a substantial improvement over purely static long positions in the underliers. These observations are broadly consistent with the findings of Fung and Hsieh (1997).

4 Gamma Required Sharpe Ratios for Hedge Funds

We may choose a level of acceptability γ^* and construct for such a level the Sharpe ratios required of funds by estimating the law of the centered and scaled random variable accessed by the fund, $F_Z(x)$ and then computing $c_Z(\gamma^*)$ the target Sharpe ratio needed for attaining the level γ^*. We performed this exercise for the 527 funds at $\gamma^* = 1.0$ for acceptability based on $MINVAR$ and $MAXVAR$. For the joint distortions of $MAXMINVAR$ and $MINMAXVAR$ we reduced the target level of γ^* to 0.75. These values for the level of acceptability are not far out of range of those observed around the .75 quantile of the levels reported in Table 2. We study the relationship between performance and other statistics of the return distribution via the required Sharpe ratio given the wide spread use of this statistic.

Of the four distortions we recall that the latter three involving reweighted expectations that exaggerate large losses, are in agreement while the first based on $MINVAR$ is relatively uncorrelated with the others. This suggests that $MINVAR$ based acceptabilities may not be the appropriate ones to use. The correlation of $MAXVAR$ based targets with the joint distortions are .9673 and .9850 while that between the two joint distortions is .9955. On the other hand the correlations of $MINVAR$ with the other three are just .0911, .3349 and .2462. We are led to recommend the construction of required Sharpe ratio targets on either of the two joint distortions. The four distortions at these acceptability levels gave average required Sharpe ratio targets of .5394, .7619, .9947 and 1.17 respectively with standard deviations of .03, .05, .05 and .07 respectively.

5 Gamma skewness and kurtosis

With a view to broadly summarizing the effects of skewness and kurtosis on required Sharpe ratios and hence the gamma, we constructed for each fund in our sample the skewness and kurtosis associated with the estimated parameters of the centered and scaled returns being accessed, in accordance with equations (7) and (8). We then regressed each of the four required Sharpe ratios on the levels of skewness and kurtosis in the distribution accessed. The results are presented in Table 3.

TABLE 3
Regression Coefficients of Required Sharpe Ratios
on Skewness and Kurtosis

	Constant	Skewness	Kurtosis	R^2
MINVAR	0.5536	0.00076	-0.0055	83.45
t-stat		(0.98)	(-7.07)	
MAXVAR	0.7572	-0.0529	-0.0037	64.08
t-stat		(-29.10)	(-2.06)	
MAXMIN	0.9954	-0.0566	-0.0062	76.54
t-stat		(-35.36)	(-3.90)	
MINMAX	1.1674	-0.0763	-0.0071	74.88
t-stat		(-35.65)	(-3.30)	

We see that for targets based on $MINVAR$ skewness has the wrong sign and is not significant. The higher R^2 is probably just a consequence of the lower variability of this target. For all the other measures we have a strongly significant and negative coefficient associated with skewness, confirming that negative skew must be compensated by higher Sharpe ratios to maintain cash flow acceptability. As noted earlier exposure to kurtosis is dominated by peakedness and hence we have a reduction in the required Sharpe ratios. Kurtosis is significant as indicated by the *t-statistics* but the effect is not as strong as that of skewness.

We separate out the effects of peakedness and tailweightedness of the distribution by regressing the required Sharpe ratios on skewness and two sepa-

rate measures for peakedness and tailweightedness. We are working with zero mean variables scaled to unit variance and for such variables one may measure peakedness by evaluating the probability of the variable being below unity in absolute value. We measure tailweightedness by computing the probability of being above 2 in absolute value. The results of this regression are provided in Table 4.

TABLE 4
Regression Coefficients of Required Sharpe Ratios
Skewness, Peakedness and Tailweightedness

	Constant	Skewness	Peakedness	Tailweight	R^2
MINVAR	1.0738	0.00832	-0.8662	2.0137	96.70
t-stat		(23.21)	(-115.69)	(30.58)	
MAXVAR	0.8121	-0.0536	-0.3494	3.889	59.01
t-stat		(-26.73)	(-8.34)	(10.56)	
MAXMINVAR	1.2864	-0.0539	-0.7447	4.8321	64.26
t-stat		(-26.41)	(-17.45)	(12.87)	
MINMAXVAR	1.4479	-0.0741	-0.7966	5.7761	64.57
t-stat		(-28.20)	(-14.52)	(11.97)	

We see from Table 4 that required Sharpe ratios are indeed negatively related to peakedness and positively related to tailweightedness for all the acceptability indices. The effect of skewness in the case of MINVAR still has the wrong sign and is now also significant. However, as commented already, this measure underweights negative skewness. For all the other stressed expectations defining acceptability we have negative and significant coefficients for skewness and peakedness and positive and significant coefficients for tailweightedness.

6 Conclusion

We apply in this paper the new performance measures developed in Cherny and Madan (2008) to the universe of hedge funds. These measures go beyond the mean and variance in assessing performance and are particularly suited to the evaluation of non-Gaussian outcomes that are typical for hedge funds. The new measures create indices of acceptability and we introduce the concept of the return gamma as the highest level of acceptability attained by an accessed return distribution. Operational procedures for constructing the gamma are implemented and illustrated for data on hedge fund returns.

Return distributions are stressed using the principle of computing expectations after a concave distortion. Four distortions are used that were introduced in Cherny and Madan (2008) and are called MINVAR, MAXVAR, MAXMINVAR and MINMAXVAR. For MINVAR the stress procedure is to draw from the distribution a number of times and take the worst outcome. For MAXVAR one takes a distribution and draws from it a number of times and the law of the best outcome must match the cash flow distribution being evaluated. The other procedures combine these stress technologies.

It is shown that target gamma levels imply a required Sharpe ratio that is responsive to the skewness, peakedness and tailweightedness of the return distribution accessed. Of the four distortions the two joint distortions are highly correlated and are the recommended distortions for evaluating performance or establishing target Sharpe ratios from both a practical and theoretical perspective.

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Appendix

Proof. of Theorem 1.

X attains the acceptability γ by condition (2) if and only if

$$\int_{-\infty}^{\infty} x \, d\left(\Psi_\gamma\left(F_X(x)\right)\right) \geq 0.$$

By construction of Z, (4) we have that

$$F_X(x) = F_Z\left(\frac{x-\mu}{\sigma}\right)$$

and hence we have acceptability just if

$$\int_{-\infty}^{\infty} x \, d\left(\Psi_\gamma\left(F_Z(\frac{x-\mu}{\sigma})\right)\right) \geq 0$$

Now make the change of variable to $z = (x - \mu)/\sigma$ to get the equivalent condition

$$\sigma \int_{-\infty}^{\infty} (\mu + \sigma z) \, d\left(\Psi_\gamma(F_Z(z))\right) \geq 0.$$

From the definition of c_Z and the fact that $\Psi_\gamma(F_Z(z))$ is another distribution function we get the result that

$$\mu - \sigma c_Z(\gamma) \geq 0,$$

or the desired lower bound on the Sharpe ratio. ∎

Proof. of Theorem 2.

This follows by noting using integration by parts that

$$c_Z(\gamma) = \int_{-\infty}^{\infty} (\Psi_\gamma(F_Z(z)) - \mathbf{1}_{z>0}) dz.$$

Hence

$$\frac{\partial}{\partial \gamma} c_Z(\gamma) = \int_{-\infty}^{\infty} \frac{\partial}{\partial \gamma} \Psi_\gamma(F_Z(z)) dz$$

and for any acceptability index it is shown in Cherny and Madan (2007) that $\frac{\partial}{\partial \gamma} \Psi_\gamma(a) \geq 0$. Differentiation once more gives the result on the second derivative. ∎